**Goal**

- Explore situations that can be modeled by an inverse variation relationship.

**Launch 3.1**

Begin with a quick review of the concept of area. Have students recall the formula for area of a rectangle, which they learned in the grade 6 unit *Covering and Surrounding*. Work a couple of examples if students need practice.

**Suggested Questions** Ask:

- **What is area?** (the number of square units that fit inside a figure)
- **How do you find the area of a rectangle?** (Multiply the length and the width.) How can you write this as a formula? \( A = lw \)
- **If you know the area of a rectangle, how can you find possible lengths and widths?** (You can find factors of the area.)

Challenge students to find the width of a rectangle given an area and a length.

- **I am thinking of a rectangle with an area of 100 square units. Its length is 8 units. What is its width?** (12.5 units)
- **How did you find this width?** (by dividing 100 by 8)
- **How does this relate to the formula for area of a rectangle?** (If the area is length times width, then width must be area divided by length.)

Introduce the context of the free lots of land in Roseville. Make sure students understand that the lots have fixed area and that they must be rectangular.

**Suggested Questions** Discuss the Getting Ready question.

- **What are some possible dimensions for a rectangular lot with area 21,800 square feet?** (Possible answers: 50 ft \( \times \) 436 ft; 100 ft \( \times \) 218 ft)

Describe the problem to students. They will look for patterns relating length and width for rectangles of fixed area.

Have students work on the problem in pairs.

**Explore 3.1**

**Listening to Students** As you observe and interact with students, consider the following questions:

- Are students aware of the division they need to do?
- Can they generalize this division to write an equation?
- Do they expect the equation to fit in the familiar form \( y = mx + b \)?
- Do they see connections between the equation here and the equation for area of a rectangle?

**Summarize 3.1**

**Suggested Questions** Ask some questions about the relationships:

- **What do you notice about how width changes in this table?** (As length increases, width decreases.)
- **How much does width decrease for each 1-inch increase in the length?** (It changes. The amount of decrease for each 1-inch increase gets smaller as the length increases.)
- **Is the change in width predictable?** (At this time, students may predict only that the change will continue to get smaller.)
- **If I gave you a length, how would you find the width?** For example, what is the width for a length of 15 inches? (Divide the area by the length, so 24 \( \div \) 15 = 1.6 inches.)
- **For rectangles with an area of 24 square inches, how could you write an equation that shows how the width depends on the length?** (Because you divide the area by the length to find the width, the equation is \( w = \frac{24}{l} \), or \( lw = 24 \) \( \div \) \( l \). Students may also write \( lw = 24 \) or \( l = \frac{24}{w} \). If so, discuss the fact that these equations are equivalent.)
• **Is this a linear relationship?** (no)

• **How can you tell?** (From the table, equal changes in length do not lead to equal changes in width. From the equation, it cannot be written as \( y = mx + b \). From the graph, the points form a curve, not a line.)

• **How are the two graphs you made similar?** (They have the same shape.)

• **How would you describe this shape?** (Students should notice that the graph is a decreasing curve.)

• **Describe the pattern of change shown in the graph.** (As the length increases, the width decreases, but not in a linear way.)

• **How are the graphs different?** (They pass through different coordinates.)

• **Where would each graph cross the y-axis?** In other words, what are the y-intercepts? (There are no y-intercepts. For example, for the first graph, the y-intercept would correspond to a rectangle with an area of 24 sq. in. and length of 0 in. This is impossible.)

• **Where would each graph cross the x-axis?** (It doesn’t. An x-intercept would represent a rectangle with a width of 0 but a positive area. This is not possible.)

This is the first time students have written equations to describe nonlinear relationships. Spend some time looking at how students found the equations. Make sure they understand how to write the equations.

• **How are the two equations you wrote alike?** (Both have the form \( w = \frac{A}{\ell} \) where \( A \) is a number that is the area of the rectangle.)

• **How are they different?** (The number \( A \) is different.)

• **Looking at the equation, how can you predict that the width will decrease as the length increases?** (You are dividing the area by the length. When you divide by a greater and greater number, the quotient gets smaller and smaller.)

Discuss other forms of the equation.

• **What is another way to write this equation?** 
  
  \((A = \ell w \text{ or } \ell = \frac{A}{w})\)

  Explain that all three equations are equivalent. You might ask students to explain how to get from one equation to another. For example, you can start with \( w = \frac{A}{\ell} \) and multiply both sides by \( \ell \) to get \( \ell w = A \). Or, you can start with \( A = \ell w \) and divide both sides by \( w \) to get \( \frac{A}{w} = \ell \).

  You might wrap up the discussion by revisiting the bridge-length data students collected in Problem 1.2.

**Suggested Question**

• **How does the data from the bridge length experiment compare with the (length, width) data for rectangles with a fixed area?** (Both sets of data show a nonlinear, decreasing pattern of change.)
Mathematical Goal

- Explore situations that can be modeled by inverse variation relationships

Launch

Review the concept of area and the formula for area of a rectangle.

- What is area?
- How do you find the area of a rectangle?
- I am thinking of a rectangle with an area of 100 square units. Its length is 8 units. What is its width?
- How did you find this width?
- How does this relate to the formula for area of a rectangle?

Introduce the context of the free lots of land in Roseville. Discuss the Getting Ready question.

Describe the problem to students and have them work in pairs.

Explore

Listen to how students are thinking about the relationship between length and width. Are they aware of the division they need to do? Can they generalize this division to write an equation? Do they see connections between the equation here and the one for area of a rectangle?

Summarize

- What do you notice about how width changes in this table?
- How much does width decrease for each 1-inch increase in the length?
- Is the change in width predictable?
- If I gave you a length, how would you find the width? For example, what is the width for a length of 15 inches?
- For rectangles with an area of 24 square inches, how could you write an equation that shows how the width depends on the length?
- Is this a linear relationship? How can you tell?
- How are the two graphs you made similar?
- Describe the pattern of change shown in the graph.
- How are the graphs different?
- Where would each graph cross the y-axis?
- Where would each graph cross the x-axis?
- How are the two equations you wrote alike? How are they different?

Materials

- Student notebooks

Materials

- Transparencies 3.1A and 3.1B
- Grid paper
- Transparencies and transparency markers

PACING 1 1/2 days
**Summarize**

- Looking at the equation, how can you predict the width will decrease as the length increases?
- What is another way to write this equation?

Explain that the different forms are equivalent. Ask students to explain how to get from one form to another.

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**ACE Assignment Guide for Problem 3.1**

**Core** 1, 2, 12

**Other Connections** 13–26; **Extensions** 40; unassigned choices from previous problems

**Adapted** For suggestions about adapting ACE exercises, see the CMP Special Needs Handbook.

**Connecting to Prior Units** 12: *Moving Straight Ahead*, Covering and Surrounding; 13–18: *Accentuate the Negative*

**Answers to Problem 3.1**

A. 1. (Figure 1)

2. Rectangles With an Area of 24 in.²

![Graph of rectangles with an area of 24 in.²](image)

3. As length increases, width decreases. The graph also decreases at a decreasing rate, so it is curved. The relationship is not linear.

B. 1. \(w = \frac{24}{\ell}\). Students may also write \(\ell w = 24\) or \(\ell = \frac{24}{w}\).

2. **Rectangles With an Area of 32 in.²**

![Graph of rectangles with an area of 32 in.²](image)

C. Both can be written in the form \(w = \frac{A}{\ell}\), where \(A\) is a fixed number. In one equation, \(A = 24\), and in the other, \(A = 32\).

D. The graphs are both decreasing curves, but they pass through different points.

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**Figure 1**

Rectangles With an Area of 24 in.²

<table>
<thead>
<tr>
<th>Length (in.)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width (in.)</td>
<td>24</td>
<td>12</td>
<td>8</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>