Goals

• Explore situations that can be modeled by inverse variation relationships
• Investigate the nature of inverse variation in familiar contexts
• Compare inverse variations with linear relationships

Students are familiar with the relationship between distance and time for travel at a fixed rate. In *Moving Straight Ahead*, students found their own walking rates and saw how distance changed as time increased. This is a linear relationship. In this problem, distance is fixed, and they see how rate varies with time.

Launch 3.2

This problem brings students from their intuitive work in Problem 3.1 to more sophisticated work, while introducing formal notation and language used with inverse variation.

The relationships you found between length and width for rectangles with an area of 24 in.$^2$ and with an area of 32 in.$^2$ are examples of an important type of nonlinear pattern called an inverse variation.

Introduce the formal definition of inverse variation:

The relationship between two variables, $x$ and $y$, is an inverse variation if $y = \frac{k}{x}$, or $xy = k$, where $k$ is a constant that is not 0.

Write the two forms of the equation on the board or overhead.

Suggested Question

• *How are these two equations related?* (They are equivalent. We could get the second from the first by multiplying both sides by $x$. Or students may notice that these equations are part of the same fact family; saying $xy = k$ is the same as saying $y = k \div x$ or $y = \frac{k}{x}$.)

Point out that for any inverse variation relationship, the product of every pair of $x$ and $y$ values is the same value, $k$. This is easy to see from the equation $xy = k$.

Revisit the inverse-variation equations students wrote in Problem 3.1. Ask students to identify the values of $k$ in those equations.

Remind students about the bridge-length experiment in Problem 1.2. Remind them that the data in that experiment were not linear. Display the data and graph from the student book, which appear on Transparency 3.2A.

Suggested Questions

• *Here is the bridge-length data one group of students collected. Is this the graph of a linear relationship?* (no)
• *How do you know?* (It’s not a straight line.)
• *Look at the table of data. Do you see any relationships or patterns?* (As length increases, the breaking weight decreases in a nonlinear pattern.)

Work with students to multiply the numbers in each (bridge length, breaking weight) pair. Write each product next to the corresponding row of the table.

• *Do you notice any patterns when we multiply each pair of values?* (The products are nearly the same.)

Discuss the first Getting Ready question.

• *Describe a curve that models the pattern in the data.* (Invite a student to the overhead to sketch a curve.)

Give students a few minutes to work in pairs to answer the rest of the questions in the Getting Ready. Then discuss their answers.

• *What value of $k$ could you use to model these data with an inverse variation equation?* (Answers will vary slightly. The average of the products, 155, is a reasonable value for $k$.)
• *Write the equation.* ($y = \frac{155}{x}$, or $xy = 155$)
• *In your equation, why does the value of $y$ decrease as the value of $x$ increases?* (The variable $x$ is in the denominator and increasing the denominator will make the fraction $\frac{155}{x}$ smaller.)
• What happens to the value of \( y \) as the value of \( x \) gets close to 0? Why is that a reasonable pattern for the bridge experiment? (As \( x \) gets close to 0, \( y \) gets very big. This is reasonable because shorter bridges are stronger.)

Move on to Problem 3.2. Remind students of a linear relationship with which they are already familiar: the relationship between distance and time for travel at a fixed speed.

**Suggested Questions**

• If you are on a car trip and driving at 60 mph, what two variables might you be interested in? (time and distance)

• What equation relates time and distance? (\( d = 60t \), where \( d \) is distance in miles and \( t \) is time in hours)

• Is this a linear relationship? (yes)

• How do you know? (Possible answer: The equation fits the form \( y = mx + b \); we have seen graphs of similar relationships, and these graphs were straight lines; the table would show an increase of 60 miles for every 1-hour increase in time.)

Tell students that you want them to think about a slightly different question.

• Suppose you are going on a car trip of 60 miles. How long will the trip take if you travel at 60 mph? (1 hour)

• What if you travel more slowly, at 30 mph? (It would take 2 hours.)

• In this problem, you are going to consider what happens if the distance, rather than the rate, is fixed. The variables will be rate and time instead of distance and time.

Have students work in pairs or groups of three.

**Explore 3.2**

As you circulate, make sure students understand that in Questions A and B, the distance is fixed. Time and average speed are changing. If you see students struggling, ask questions that point their attention to this idea, and remind them of the differences between these problems and the more familiar linear relationship in which speed is fixed.

Make sure all students can explain why the relationships in Questions A and B are inverse variations. Even students who are struggling with the ideas should be able to point to the shape of the graphs as justification. Most students should recognize the form of the equation by this point. If this is not the case, you will want to spend time in the summary with this idea.

**Summarize 3.2**

Spend some time with students discussing how they found the equations in Questions A, B, and C. Make sure they understand how to write the equations.

• How are the equations you found in Questions A and B different from the equation you found in Question C? (The equation in Question C is linear; the other two are not.)

• What do we know about the relationships among time, speed, and distance? [There are different ways to think about it: Distance equals speed times time \( (d = st) \), speed equals distance divided by time \( (s = \frac{d}{t}) \), and time equals distance divided by speed \( (t = \frac{d}{s}) \).]

• What are some differences in the graphs, tables, and equations of linear relationships and those of inverse variations? (The graph of a linear relationship is a straight line, which may be increasing or decreasing. The graph of an inverse variation is a decreasing curve. The table for a linear relationship shows a constant rate of change. For an inverse variation, the rate of change is not constant; the \( y \)-values decrease at a decreasing rate as \( x \) increases. The equation for a linear relationship can be written in the form \( y = mx + b \). The equation for an inverse variation can be written as \( y = \frac{k}{x} \).)
3.2 Bridging the Distance

Mathematical Goals
- Explore situations that can be modeled by inverse variation relationships
- Investigate the nature of inverse variation in familiar contexts
- Compare inverse variations with linear relationships

Launch
Introduce inverse variations. Relate the equations from the last problem to the general equations and have students identify the values of \(k\).

Remind students of the Problem 1.2 experiment. Display Transparency 3.2A.
- Is this the graph of a linear relationship? How do you know?
- Look at the table of data. Do you see any relationships or patterns?
- Do you notice any patterns when we multiply each pair of values?

Have students work on the Getting Ready questions, and then discuss them.

Introduce the context of Problem 3.2.
- If you are on a car trip and driving at 60 mph, what two variables might you be interested in? What equation relates time and distance?
- Is this a linear relationship? How do you know?
- Suppose you are going on a car trip of 60 miles. How long will the trip take if you travel at 60 mph? What if you travel more slowly, at 30 mph?
- In this problem, you are going to consider what happens if the distance, rather than the rate, is fixed. The variables will be rate and time instead of distance and time.

The class can work in pairs or groups of three.

Explore
As you circulate, make sure students understand that in Questions A and B, the distance is fixed. Time and average speed are changing.

Make sure all students can explain why this relationship is an inverse variation.

Summarize
- How are the equations you found in Questions A and B different from the equation you found in Question C?
- What do we know about the relationships among time, speed, and distance?
- What are some differences in the graphs, tables, and equations of linear relationships and those of inverse variations?
**ACE Assignment Guide for Problem 3.2**

**Core** 3–9, 28

**Other Connections** 27, 29–31; **Extensions** 41–45; unassigned choices from previous problems

**Adapted** For suggestions about adapting ACE exercises, see the *CMP Special Needs Handbook*

**Connecting to Prior Units** 27–28: *Data About Us*; 30–31: *Moving Straight Ahead*

### Answers to Problem 3.2

**A. 1.**

Cordova’s Baltimore Trips

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>1.5</th>
<th>10</th>
<th>14</th>
<th>4</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Speed (mi/h)</td>
<td>333.3</td>
<td>50</td>
<td>35.71</td>
<td>125</td>
<td>27.78</td>
</tr>
</tbody>
</table>

2. Curves will vary. This graph includes the points added in part (5).

As trip time increases, average speed decreases at a decreasing rate, causing a curved pattern of points. This is an inverse variation pattern.

3. \( s = \frac{500}{t} \) (or \( st = 500 \) or \( st = \frac{500}{x} \))

4. 6 hr: 83.33 mph; 8 hr: 62.5 mph; 12 hr: 41.67 mph; 16 hr: 31.25 mph

5. See the graph for the points. Fit will vary.

**B. 1.** 300 mi. Multiply the average speed by the trip time for any average speed.

2. \( t = \frac{300}{s} \) (or \( s = \frac{300}{t} \) or \( st = 300 \))

3. As average speed increases, travel time decreases at a decreasing rate, forming a curve. In the equation \( t = \frac{300}{s} \), we are dividing by \( s \), and as the divisor \( s \) increases, the quotient \( t \) decreases.

4. 6.67 hr (or 6 hr 40 min) and 4.61 hr (or 4 hr 37 min)

**C. 1.**

Trip to Mackinac Island

<table>
<thead>
<tr>
<th>Travel Time (h)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (mi)</td>
<td>0</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>200</td>
<td>250</td>
<td>300</td>
</tr>
</tbody>
</table>

As trip time increases, average speed decreases at a decreasing rate, causing a curved pattern of points. This is an inverse variation pattern.

2. \( d = 50t \)

3. The distance changes in an increasing linear pattern. The constant rate of change is 50 mph.

4. The *(time, distance)* relationship has a straight-line graph and an equation of the form \( y = mx + b \). The other relationships are inverse variations. Their graphs are decreasing curves, and their equations are of the form \( y = \frac{k}{x} \).